S520 Homework 3

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January 18, 2012

Chapter 4, Section 5, #1:

(a) The pmf of X is $f(y) = P(x = y)$, for $y \in X(S)$ and 0 otherwise, i.e.:

$$f(1) = P(X = 1) = 0.1 = P(X = 6) = f(6)$$

$$f(3) = P(X = 3) = 0.4 = P(X = 4) = f(4)$$

and 0 otherwise

(b) The cdf of X is $F(y) = P(X \leq y)$, for $y \in R$, i.e.

$$F(k) = 0, -\infty \leq k < 1$$

$$F(1) = F(X \leq 1) = 0.1 \ ; \ F(3) = F(X \leq 3) = 0.5$$

$$F(4) = F(X \leq 4) = 0.9 \ ; \ F(6) = F(X \leq 6) = 1 \ ; \ F(h) = 1, h \geq 6$$

(c) Expected value of X:

$$EX = \sum_{x \in X(S)} xf(x) = 1 \cdot f(1) + 3 \cdot f(3) + 4 \cdot f(4) + 6 \cdot f(6)$$

$$= 1 \cdot 0.1 + 3 \cdot 0.4 + 4 \cdot 0.4 + 6 \cdot 0.1 = 0.1 + 1.2 + 1.6 + 0.6 = 3.5$$

(d) Variance of X:

$$VarX = EX^2 - (EX)^2 = 1^2 \cdot f(1) + 3^2 \cdot f(3) + 4^2 \cdot f(4) + 6^2 \cdot f(6) - 3, 5^2$$

$$= 0.1 + 3.6 + 6.4 + 3.6 - 12.25 = 13.7 - 12.25 = 1.45$$

(e) Standard Deviation of $X = \sqrt{VarX} = \sqrt{1.45} = 1.204159458$

Chapter 4, Section 5, #5:

(a) $P(A = 1, B = 3, C = 4, D = 6) =$ events are independent, thus $= P(A = 1) \cdot P(B = 3) \cdot P(C = 4) \cdot P(D = 6) = 0.1 \cdot 0.4 \cdot 0.4 \cdot 0.1 = 0.0016$

(b) Same as before but in different order, i.e., $P(A = 1, B = 6, C = 3, D = 3) =$ events are independent, thus $= P(A = 1) \cdot P(B = 6) \cdot P(C = 3) \cdot P(D = 4) = 0.1 \cdot 0.1 \cdot 0.4 \cdot 0.4 = 0.0016$

(c) There are 4! ways of getting different results for each of these four astragali. Thus, the probability of producing a venus is: $4! \cdot 0.0016 = 0.0384$

(d) A run of a venus is getting a venus each time in the run. The key here is to regard each venus as independent of another. Thus, the probability of producing two venuses one after another is $0.0016 \cdot 0.0016$. Generalizing this result, the probability of producing k venuses in a run of length k is 0.0016^k.

Chapter 4, Section 5, #10: On the one hand, let Y = number of people that accepted the invitation. Then

$$Y \sim Binomial(12, 0.5)$$

On the other, let Z = number of people that accepted the invitation and actually show to the dinner party. Then,

$$Z \sim Binomial(k, 0.8), \text{where } k \text{ will vary according to the number of attendants}$$

It is important to note that I am assuming that if a guest does not accept the invitation, he/she does not show to the party.

Now, we want to know what is the probability that more guests attend than can be accommodated. This means that either 8 guests accepted the invitation and 8 attended, or 9 guests accepted and 8 attended, or 9 accepted and 9 attended, ..., or 12 accepted and 12 attended.

In the model this is equal to

$$\sum_{j=8,\ 8\leq j\leq k}^{12} P(Y=j \text{ and } Z=k) = \sum_{j=8,\ 8\leq j\leq k}^{12} P(Y=j)\cdot P(Z=k)$$

Assuming that Y and Z are independent:
$P(Y=8)\cdot P(Z=8)+P(Y=9)\cdot P(Z=8)+P(Y=9)\cdot P(Z=9)+P(Y=10)\cdot P(Z=8)+P(Y=10)\cdot P(Z=9)+P(Y=10)\cdot P(Z=10)+P(Y=11)\cdot P(Z=8)+P(Y=11)\cdot P(Z=9)+P(Y=11)\cdot P(Z=10)+P(Y=11)\cdot P(Z=11)+P(Y=12)\cdot P(Z=8)+P(Y=12)\cdot P(Z=9)+P(Y=12)\cdot P(Z=10)+P(Y=12)\cdot P(Z=11)+P(Y=12)\cdot P(Z=12)$.

Using R, I obtained the following probability: 0.05730992. Here is the function I wrote for this problem:

```
hw3 <- function(y){
for (i in 8:12){
for(j in 8:i){
#Y variable - > print(dbinom(i,12,0.5))
#Z variable - > print(dbinom(j,i,0.8))
y <- y + (dbinom(i,12,0.5) * dbinom(j,i,0.8))
}
```

```
}
print("Result")
print(y)
}
```

Chapter 4, Section 5, #12:

(a) Let Y = number of students that will need to be accommodated. Then:

$$Y \sim Binomial(225; 0.36)$$

The number of expected freshmen that will have to be accommodated are equal to $EY = 225 \cdot 0.36 = 81$

(b) $P(Y > 95) = 1 - P(Y \leq 95) =$ (using R) $= 1 - pbinom(95, 225, .36) = 0.02291658$

Chapter 4, Section 5, #14:

(a) Let Y = number of symbols correctly identified. Then.

$$Y \sim Binomial(25, \frac{1}{5})$$

We should expect the receiver to identify $EY = 25 \cdot \frac{1}{5} = 5$ symbols correctly.

(b) $P(Y > 7) = 1 - P(Y \leq 7) =$ (using R) $= 1 - pbinom(7, 25, 1/5) = 0.1091228$

(c) Each receiver can be modeled as a binomial variable:

$$Y_i = \text{ score of receiver } i \sim Binomial(25; 1/5)$$

Each Y_i is i.i.d and $1 \leq i \leq 25$. The probability we want is:

$P(Y_1 > 7 \text{ or } Y_2 > 7 \text{ or... or } Y_{20} > 7) = 1 - P((Y_1 > 7 \text{ or } Y_2 > 7 \text{ or... or } Y_{20} > 7)^c) =$

$1 - P(Y_1 \leq 7 \text{ and } Y_2 \leq 7 \text{ and... and } Y_{20} \leq 7) = 1 - P(Y_1 \leq 7) \cdot P(Y_2 \leq 7) \cdot ... \cdot P(Y_{20} \leq 7) =$
$1 - (pbinom(7, 25, 1/5))^{20} = 1 - 0.8908772^{20} = 0.900835341$

Chapter 4, Section 5, #15:

Let Y_i = the number of heads counted by student i. Then

$$Y_i \sim Binomial(89; 0.3), \text{for each } 1 \leq i \leq 1500$$

We want to calculate the following probability:

$$P(\text{at least one student observe no more than two heads}) =$$

$$1 - P(\text{none of the students observe no more than two heads}) =$$

$$1 - \prod_{i=1}^{1500} P(Y_i \leq 2) = 1 - pbinom(2, 89, 0.3)^{1500} \approx 1 - 0 = 1$$